The John Moore Primary School
A Progression of Calculation Methods.

Introduction.

At the John Moore Primary School, children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas of addition, subtraction, multiplication and division they develop ways of recording these calculations using particular methods, signs and symbols.

Over time, children learn to use a range of recording methods, such as number lines to support their mental and informal written methods. As their mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient and standard written methods. By the end of Year Six children have been introduced to mental, written and calculator methods that they can use to solve the four number operations. They are then able to decide which method is most appropriate for a particular calculation, to use that method and have strategies to check its accuracy.

At whatever stage in their learning, and whatever method is being used, it is vital that children have a secure and appropriate knowledge of number facts. Therefore, at The John Moore Primary School our aim is that when children leave they:

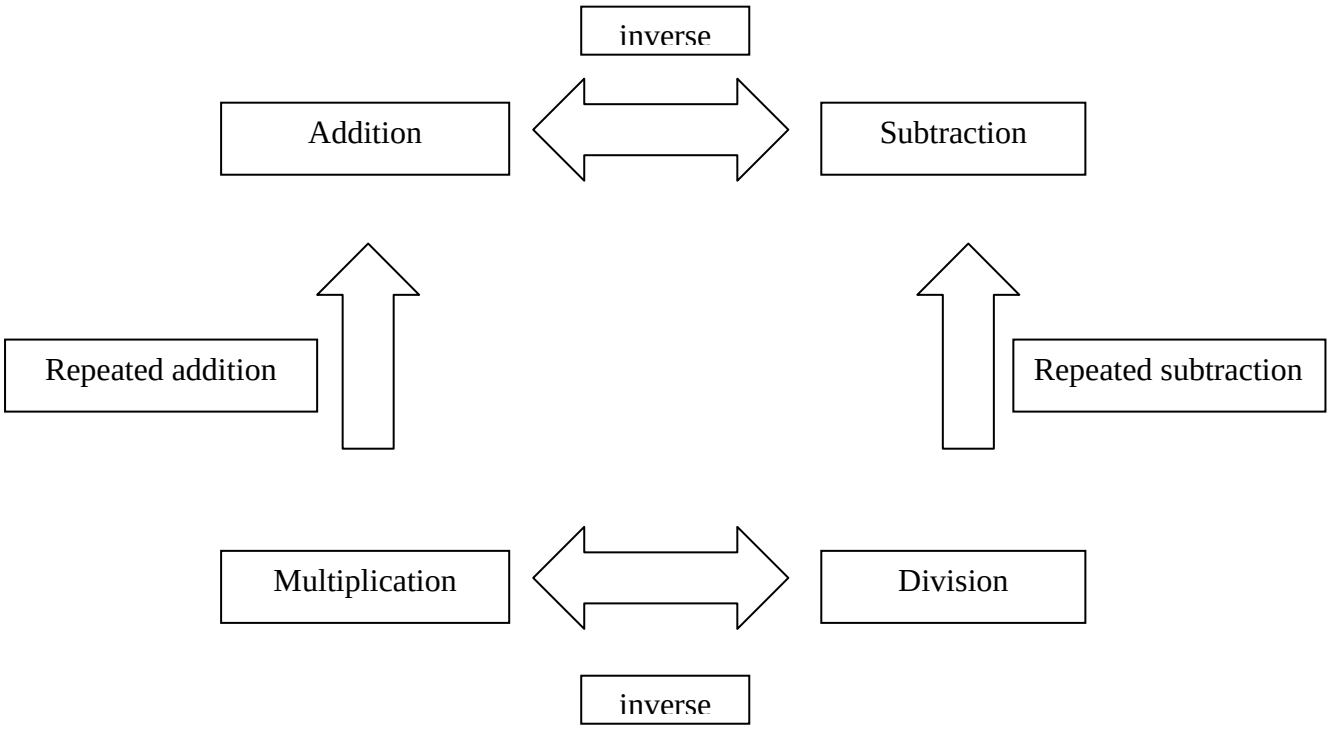
- Have a secure knowledge of number facts and a good understanding of the four operations
- Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving larger numbers
- Make use of diagrams and informal notes, 'jottings', to help record steps and part answers when using mental methods
- Have an efficient, reliable, compact written method of calculation for each operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally
- Use a calculator effectively, using their mental skills to monitor the process and check for sense and accuracy.

Mental methods of calculation.

Oral and mental work in mathematics is essential, as early practical, oral and mental work lays the foundations for later written methods. By providing children with a good understanding of how the four operations build on counting strategies and a knowledge of place value and number facts, children will recognise how the operations relate to one another and how the rules of arithmetic can be used and applied.
Ongoing oral and mental work provides practise and consolidation of these ideas.

We aim to provide all children with a secure knowledge of numbers i.e.

- Recall number facts instantly – addition and subtraction facts to 10 (Year 2), sums and differences of multiples of 10 (Year 3) and multiplication facts up to 10 x 10 (Year 4).
- Use taught strategies to work out a calculation – for example, recognise that addition and multiplication could be done in any order but that subtraction and division cannot.
- Understand and use the term 'inverse' – i.e. that addition and subtraction are inverse operations and multiplication and division are inverse operations.



Written methods of calculation.

As children progress through the school they move from informal methods of recording to expanded methods and then to a compact written method for each of the four operations.

The aim is that by the end of Key Stage Two, the majority of children should be able to use an efficient written method for each operation with confidence and understanding. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator.

Written methods for addition.

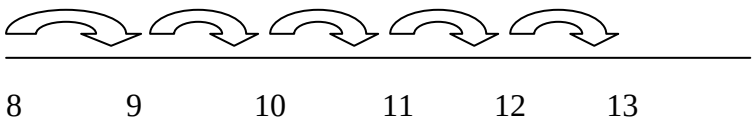
The aim is that children use mental methods when appropriate, but for calculations they cannot do in their heads they use an efficient written method accurately and with confidence.

To add successfully, children need to be able to:

- Recall all addition pairs to 10 + 10
- Add mentally a series of one-digit numbers, such as 5+8+4
- Add multiples of 10 (60+70) or 100 (600+700) using knowledge of addition facts (6+7) and place value
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1

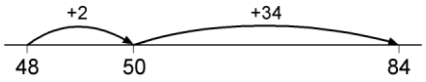
Reception.
Here the emphasis is on practical and oral methods. Children will make and count sets of objects and combine them. They may record their work using a number sentence, such as 3+5=8. Children are introduced to the addition and equals signs.

Year One.
Here children will continue to work in a very practical way and may start to use number lines to record their calculations.
8+5=13

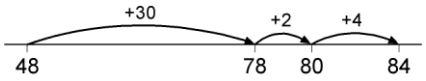


Year Two.
Here children continue to use number lines, but progress to adding in ‘jumps’ of numbers. This is especially so when ‘bridging’ from one multiple of 10 to another.
8+7=15
The idea is to add to the next multiple of 10 and then add anything that is left over. With the example below 2 is added to reach 10 and then the remaining 5 is added

48+36=84
First add 2 to reach 50 then add the remaining 34.



Another way of adding using a number line is to add the tens first. The above calculation would therefore be completed as below.
48+30=78+2=80+4=84



Partitioning is also introduced in Year Two as a means of adding two-digit numbers. Partitioning involves using place value to split two-digit numbers into the 'tens' and 'ones' that they are made up of and then to 'recombine' them to find the total.
47+76=
40+70+7+6=
110+13 =123

Year Three.
Children continue to use partitioning in Year Three and extend it to three-digit numbers and beyond.
123+245=
100+200+20+40+3+5=
300+60+8=368

They also start to transfer this method into a column method where ones are placed under ones and tens under tens etc.

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +\,76 & & 70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$

Year Four.
The expanded column method used in Year Four is an extension of the above where the tens and ones are added separately. Children can either add the tens or ones first but as they gain confidence they are encouraged to start by adding the ones digit first.
Adding the tens first:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 110 \\ \ 13 \\ \hline 123 \end{array}$$

Adding the ones first:

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline \ 13 \\ 110 \\ \hline 123 \end{array}$$

Year Five.
The children move from the expanded method to the compact method in Year Five. In this method, recording is reduced further. Digits are 'carried' below the line.

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ {\scriptstyle 1\,1} \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ {\scriptstyle 1\,1} \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ {\scriptstyle 1\,1} \end{array}$$

Year Six.
Children continue to use the compact column addition method but extend to larger whole numbers and decimals.

$$\begin{array}{r} 5.2 \\ +6.6 \\ \hline 11.8 \end{array}$$

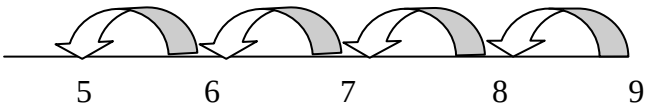
Written methods for subtraction.

To subtract successfully, children need to be able to:

- Recall all addition and subtraction facts to 20
- Subtract multiples of 10(such as 160-70) using the related subtraction fact 16-7, and their knowledge of place value.
- Partition two-digit and three-digit numbers into multiples of hundred, ten and one in different ways (e.g. partition 74 into 70+4 or 60+14)

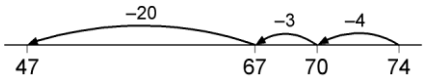
Reception.
Here the emphasis is on practical and oral methods. Children count sets and objects and then 'take away' one or more. They use vocabulary such as 'less than' and record their work using number sentences such as 8-1=7. They are introduced to the subtraction sign.

Year One.
Again, most work is practical and oral although children will progress to finding 10 less as well as 1 less using their knowledge of place value. They will again record using number sentences and jump back in ones on number lines.
9-4=5

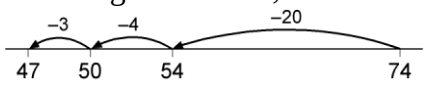


Year Two.
As with addition, children in Year Two start to use number lines to record or explain the steps of their mental calculations. Again the use of bridging is involved and children use the idea of 'counting back'.
15-7=8
Here 15-5 is done first to reach the multiple of 10 and then the remaining 2 are subtracted.

A calculation like 74-27 can be recorded by counting back using bridging:



or by subtracting the 20 first, followed by the 7:



The children may also start to be introduced to the ideas of 'counting up' and partitioning as shown for Years Three and Four.

Year Three.

In Year Three the children are shown how to use a counting up method and partitioning to help transfer their jottings from number lines to column methods. Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally.

For 74-27 this involves partitioning the 27 into 20 and 7. Some children may need to partition the 74 into 70 and 4 or 60 and 14 to help them carry out the subtraction.

74 – 27 =

74 – 20 – 7 =

54 – 7 = 47

or

74 – 27 =

70 + 4 – 20 – 7 =

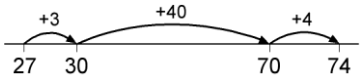
60 + 14 – 20 – 7 =

40 + 7

Year Four.

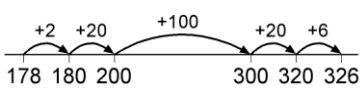
The mental method of counting up from the smaller number to the larger number involves the children understanding that subtraction and addition are inverse calculations. This can be recorded on number lines or vertically in columns.

74-27



$$\begin{array}{rl} 74 & \\ -\ \underline{27} & \\ 3 & \rightarrow 30 \\ 40 & \rightarrow 70 \\ \underline{4} & \rightarrow 74 \\ 47 & \end{array}$$

This method can also be extended to three-digit numbers.



$$\begin{array}{rl} 326 & \\ -\underline{178} & \\ 2 & \rightarrow 180 \\ 20 & \rightarrow 200 \\ 100 & \rightarrow 300 \\ \underline{26} & \rightarrow 326 \\ 148 & \end{array}$$

Year Five.

Here children are introduced to the expanded layout of subtraction that mirrors the addition column method. They use partitioning to create an expanded layout, which then leads to a compact method. The amount of time spent teaching and practising the expanded method will depend on how secure children are in their recall of number facts and with partitioning. The aim is that all can use the compact method by the end of Year Five.

Here the first calculation shows the partitioning involved, the second shows the expanded method and the third shows the compact method.

74-27=

$$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 60\quad 14} \\ \cancel{70}+\cancel{4} \\ -\ 20+7 \\ \hline 40+7 \end{array} \qquad \begin{array}{r} {\scriptstyle 6\ 14} \\ \cancel{7}\ \cancel{4} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

741-367=

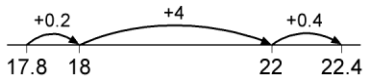
$$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 600\quad 130\quad 11} \\ \cancel{700}+\cancel{40}+\cancel{1} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array} \qquad \begin{array}{r} {\scriptstyle 6\ 13\ 11} \\ \cancel{7}\ \cancel{4}\ \cancel{1} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

As with addition, children start by subtracting the ones, then tens etc. Sometimes adjustment or decomposition is needed. This is sometimes called 'borrowing'.

Year Six.

Children continue to use column subtraction, either working with the expanded or compact method, depending on their confidence.

This is extended to include larger numbers, decimals and examples where quite complex adjustment of hundreds, tens and units are involved.



$$\begin{array}{rl} 22.4 & \\ -17.8 & \\ \hline 0.2 & \rightarrow 18 \\ 4.0 & \rightarrow 22 \\ 0.4 & \rightarrow 22.4 \\ \hline 4.6 & \end{array}$$

Here both the tens and ones digits to be subtracted are larger than the starting digit so all columns need adjustment.

563-278=

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+150+13 \\ -\ 200+\ 70+\ 8 \\ \hline 200+\ 80+\ 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 400\quad 150\quad 13} \\ \cancel{500}+\cancel{60}+\cancel{3} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 15\ 13} \\ \cancel{5}\ \cancel{6}\ \cancel{3} \\ -\ 2\ 7\ 8 \\ \hline 2\ 8\ 5 \end{array}$$

Here 0 acts as a place holder for the tens and the adjustment must be done in two stages.

503-278=

$$\begin{array}{r} 500+\ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ 8 \\ \hline 200+20+\ 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 400\quad 90\quad 13} \\ \cancel{500}+\cancel{0}+\cancel{3} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 9\ 13} \\ \cancel{5}\ \cancel{0}\ \cancel{3} \\ -\ 2\ 7\ 8 \\ \hline 2\ 2\ 5 \end{array}$$

Written methods for multiplication.

To multiply successfully, children need to be able to:

- Recall all multiplication facts to 10x10
- Partition numbers into multiples of one hundred, ten and one
- Work out products such as 70x5, 70x50, 700x5, 700x50 using the related fact 7x5 and their knowledge of place value
- Add two or more single-digit numbers mentally
- Add multiples of 10 or 100 using the related addition fact and their knowledge of place value.

Reception and Year One.
Children are introduced to the concept of multiplication through practical activities that involve counting in steps larger than one. For example in Reception may count in twos by counting pairs of socks and this continues in Year One where they will count in twos tens and also fives.

Year Two.
In Year Two children explore the concept of multiplication in the contexts of doubling numbers (2 times table) and making groups and sets of objects of different sizes. They may record their practical work as repeated addition or as an array. They will also be introduced to the multiplication sign.
For example:
3 groups of 4 stars can be recorded as

4+4+4=12 or
3+3+3+3=12
3x4=12
4x3=12

Year Three.
Here children continue to use arrays and start to use partitioning and brackets to help them record multiplication calculations. By partitioning two-digit numbers into tens and ones, these can be multiplied separately to form partial products. These can then be recombined to find the total product.

$$14 \times 3 = (10 + 4) \times 3$$
$$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$$

An expanded method, which uses a grid to show the same steps, is also introduced in Year Three.

X	3
10	30
4	12

42

Year Four.

Children continue to practise the grid method and extend it to use for TU x TU and HTU x TU

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

Year Five.

In Year Five short multiplication is introduced. The grid method is developed into an expanded form of short multiplication using a column format, but showing the working.

TU x U

×	7
30	210
8	56
	266

$$\begin{array}{r} 30+8 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

$30 \times 7 = 210$

$8 \times 7 = 56$

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

TU xTU

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 1000 \\ 120 \\ 350 \\ 42 \\ \hline 1512 \\ 1 \end{array}$$

$50 \times 20 = 1000$

$6 \times 20 = 120$

$50 \times 7 = 350$

$6 \times 7 = 42$

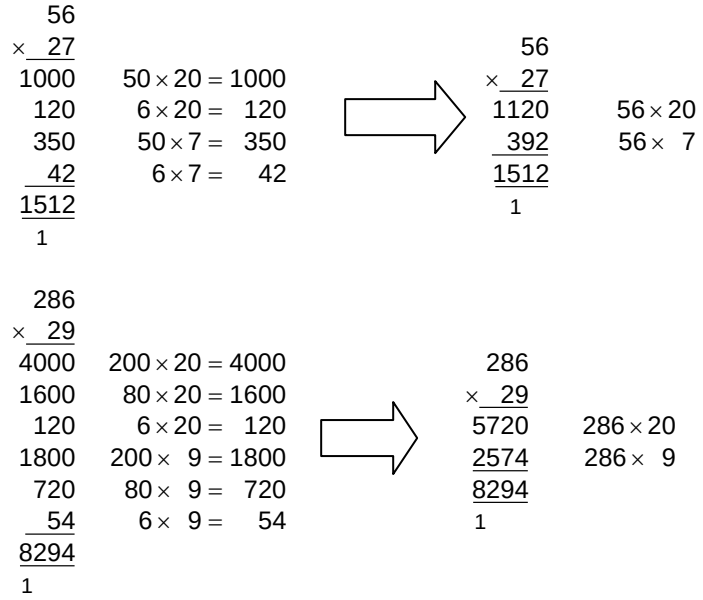
Year Six.

Here the recording is reduced further to produce a compact short multiplication method by recording 'carried' digits under the line.

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 210 \\ 56 \\ \hline 266 \end{array}$$

$$\begin{array}{r} 38 \\ \times\ \ 7 \\ \hline 266 \\ 5 \end{array}$$

This can also be extended to TU xTU and HTU x TU



Written methods for division.

To divide successfully in their heads, children need to be able to:

- Understand and use the vocabulary of division
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1
- Recall multiplication and division facts to 10 x 10
- Know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5.
- Understand and use multiplication and division as inverse operations.

To carry out written methods of division successfully, children also need to be able to:

- Understand division as repeated subtraction
- Estimate how many times one number divides into another
- Multiply a two-digit number by a single-digit number mentally
- Subtract numbers using the column method.

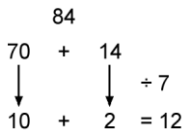
Reception, Year One and Year Two.
Children are introduced to the concept of division through practical activities that involve sharing, grouping and making sets including pairs. In Year Two children will use equipment to share and group and record using pictures. They will explore halving numbers and note how division is the opposite (inverse) of doubling. Halving is viewed as splitting into two equal parts, therefore dividing by 2. This is also extended to finding quarters or dividing by 4. Children will be familiar with the division sign.

Year Three.
Here children explore the idea of division as repeated subtraction and start to use fractions to split a larger number into smaller groups.

For example:
100÷2=50
100-50-50=0
½ of 100=50

Children will explore fractions of ½, ¼ and 1/10.

Year Four.
In Year Four children start to record their division calculations more formally. The written methods at this stage reflect the processes used in their mental calculations, especially partitioning and repeated subtraction.
For example:
84÷7=



or

$64 \div 4 = (40 + 24) \div 4$
$= (40 \div 4) + (24 \div 4)$
$= 10 + 6 = 16$

Children can use the concept of repeated subtraction and combine it with the column method of recording used for addition and subtraction. This expanded method is often called ‘chunking’ as it takes way chunks or multiples of the divisor. This method can also be used when number are not completely divisible but leave remainders.
For example:
97 ÷ 9

$9\overline{)97}$
$-\ 90 \quad 9 \times 10$
7
Answer: 10 R 7

Year Five.
Through Year Five children continue to explore ‘chunking’ and develop its use for HTU ÷ TU and ThHTU ÷TU.
For example:
196÷6=

$6\overline{)196}$
$-\ 60 \quad 6 \times 10$
136
$-\ 60 \quad 6 \times 10$
76
$-\ 60 \quad 6 \times 10$
16
$-\ 12 \quad 6 \times 2$
$4 \quad 32$
Answer: 32 R 4

Year Six.

By Year Six, children should be secure in their knowledge and skills when partitioning and using repeated subtraction. Therefore they move from the expanded method to a more concise short division method or long division method.

Short division is a more compact recording of the mental method of partitioning.

For example:

For TU÷U=

$81 \div 3 = (60 + 21) \div 3$
$= (60 \div 3) + (21 \div 3)$
$= 20 + 7$
$= 27$

$\Rightarrow \quad 3\overline{)60+21} \; 20+7 \quad \Rightarrow \quad 3\overline{)8^{2}1} \; 27$

For HTU÷U=

$291 \div 3 = (270 + 21) \div 3$
$= (270 \div 3) + (21 \div 3)$
$= 90 + 7$
$= 97$

$\Rightarrow \quad 3\overline{)290+1} = 3\overline{)270+21} \; 90+7 \quad \Rightarrow \quad 3\overline{)2\,9^{2}1} \; 97$

Long division is a more compact recording that links to chunking and is used to tackle HTU÷TU=

For example:

$24\overline{)\,560}$

$20 - \underline{480} \qquad 24 \times 20$
$\quad 80$
$3 \quad \underline{72} \qquad 24 \times 3$
$\quad 8$

Answer: 23 R 8

or

$24\overline{)\,560}$ $\quad 23$

$-\underline{480}$
$\quad 80$
$-\underline{72}$
$\quad 8$

Answer: 23 R 8

J Hannett April 2007.
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